MATH 464 (THEORY OF PROBABILITY)
PRACTICE FOR CHAPTER 8

FALL 2017

(1) Let X be any continuous random variable. For any $t > 0$, prove that

$$\mathbb{P}(X > y) \leq e^{-ty}\ \mathbb{E}(e^{tX}).$$

(2) A basketball player makes 80% of his free throws on the average. The player attempts free throws repeatedly until he makes 25. What is the probability that at least 29 throws will be necessary?

(3) Suppose 1% of all screws made by a machine are defective. We are interested in the probability that a batch of 225 screws has at most one defective screw. Compute this probability by
   (a) Poisson approximation.
   (b) normal approximation.

   Note that the exact value of this probability is 0.34106. Which approximation works better when p is small and n is large?

(4) Let $X_1, X_2, \ldots, X_n$ be a sequence of independent standard normal random variables. Let $S_n = X_1^2 + \ldots + X_n^2$. Find the mean value and variance of S_n. Describe the normal approximation to S_n. Use it to estimate

$$\mathbb{P}(S_n \leq n + \sqrt{2n}).$$

   Note: S_n is said to have χ^2-distribution with n degrees of freedom.